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THE TEACHING OF MATHEMATICS TO STUDENTS OF ENGINEERING[1]

FROM THE STANDPOINT OF THE PRACTISING ENGINEER

I am honored by being asked to say a few words to you about the results of my experience as to the needs of the teaching of mathematics to students of engineering from the point of view of a practical engineer. I have had the good fortune of receiving quite a thorough mathematical training in the École des Ponts et Chaussèes of France, and I have also had the good fortune of developing into a fairly practical engineer; my remarks will therefore be backed by actual experience.

Mathematics is to an engineer what anatomy is to a surgeon, what chemistry is to an apothecary, what the drill is to an army officer. It is indispensable. I think we all agree on this point.

There is a considerable agitation at this time in France and Germany, especially the former, favoring the limitation of the

[1] What is Needed in the Teaching of Mathematics to Students of Engineering? (*a*) Range of Subjects; (*b*) Extent in the Various Subjects; (*c*) Methods of Presentation; (*d*) Chief Aims. A series of prepared discussions following the formal presentation of the subject by Professor Edgar J. Townsend, Professor Alexander Ziwet, Mr. Charles F. Scott and President Robert S. Woodward. (See SCIENCE, July 17, 1908, pp. 69–79; July 24, 1908, pp. 109–113, and July 31, 1908, pp. 129–138.) Presented before Sections D and A of the American Association for the Advancement of Science and the Chicago Section of the American Mathematical Society, at the Chicago meeting, December 31, 1907.

present mathematical program of the engineering schools on the ground that it is unnecessarily extensive. From personal observation, I can say that the program there covers a considerably wider range than in the average American college. In the first place, a student entering an engineering college on the European continent must already know the analytical geometry, the descriptive geometry, the rudiments of differential and integral calculus, none of which are taught here until the student enters college. The average length of a college engineering course abroad is four years, one of the exceptions being the École Centrale, of Paris, France, where the course is only three years, but where the entering examinations are of a comparatively high standard and the students must be above the average in ability and application in order to hold their own during the college course. It is obvious, therefore, that in American colleges, time is spent on pure mathematics which could be devoted to practical study. I believe the time will come when only applied mathematics will be taught in colleges, and all necessary abstract mathematics will form a part of the conditions for entering.

As time goes on, every profession tends more and more towards specialization. This tendency is quite marked in the engineering profession. It would take too long to enumerate all of these special branches of engineering, but nearly every branch demands a somewhat different mathematical training. The time may come when this specialization will extend over the study of abstract mathematics, differing with each student according to the branch of engineering he intends to follow. For instance, a railway engineer who may aspire to become a railroad official requires less knowledge of calculus than an electrical or a bridge engineer; on the other hand, he requires a greater knowledge of geology than the electrical engineer, and a greater knowledge of common law than the bridge engineer. As my remarks are merely intended to furnish topics for discussion, I will put the following question: In view of the fact of the steadily growing scope of special education will it be desirable and possible to specialize mathematical courses in colleges and adapt them to each branch of engineering? This, as I understand, is done at present only to a small extent in applied mathematics.

Bridge engineering, of which I have made a specialty, requires probably as high a mathematical training as any other branch of the profession, and yet, I find that part of the higher mathematics which I have studied in college, apart from the drilling features of such studies, has been entirely useless; for instance, the theory of differential equations. The time I spent on it, though considerable, was not sufficient to make me understand it thoroughly, and would have been better employed in the study of the methods of least work, for instance, which no bridge engineer should neglect to study.

On perusing the elementary books used in high schools, I have been often struck with the dry, uninteresting manner in which the various subjects are being treated. The examples are mostly abstract, very few practical problems to work out. Unless the student is very intelligent, his mind retains nothing beyond a chaos of formulæ hard to remember and a few mechanical means of solving abstract problems. He is incapable of applying an equation to a practical problem. The methods of presentation should, therefore, be such that the student knows the why and wherefore of each operation—in other words, that he learns to *think mathematically*. This training in mathematical thinking should

also be the chief aim: one does not know a foreign language unless one is able to think in that language; one does not know mathematics unless one is able to think mathematically. It is not necessary for that to go up into the highest mathematics, but it is necessary to be thoroughly drilled in elementary principles of each subject. These elementary principles should become a second nature to the student, just as a language becomes a second nature when it is thoroughly acquired. Problems arise every day in the practise of an engineer, which a mathematical mind can solve without going into calculations, such principles as those of maxima and minima, those of least work, of cumulative effect of forces and others are invaluable in assisting to arrive at a logical solution of many problems without the use of a scrap of paper; but in order that they may be applied, one has to be able to think mathematically. With a proper foundation, the engineer's mind becomes so trained that he applies those fundamental principles unconsciously; they direct his line of thought automatically, so to speak. How to secure such a foundation in a student must be left to those who make a life-study of teaching.

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The methods of teaching mathematics to engineering students in vogue twenty years or more ago, while often sufficiently strenuous, were invariably far from satisfactory, in that they failed to show the application of the subjects to engineering practise and to explain that mathematical quantities represent something real and tangible, not merely abstractions. Possibly methods have changed of late years; but nothing that the writer has seen or heard indicates to him that any fundamental improvement has been effected. Most people continue to believe that mathematical subjects are taught mainly for the purpose of training the mind, and that the manipulations involved in this branch of science are simply mental gymnastics. Moreover, even among engineers and professors, only a few recognize adequately the great importance of mathematics in engineering and that it is something real and substantial instead of fictitious and imaginary. It is true that higher powers than the third are not conceivable entities; but the mathematician recognizes them as temporary multiples for future reduction to entities.

The engineering student in his pure-mathematical classes is not taught what equations really mean, nor what are their denominations or those of their component parts. All that he learns is how to juggle with quantities in order to produce certain results. It is left to the professor of rational mechanics to teach engineering students the reality of mathematics; and too often he fails to do so, sometimes, perhaps, because his own conception thereof is rather vague.

Concerning the teaching of pure mathematics by the professor of rational mechanics the writer speaks from personal experience; for more than a quarter of a century ago he taught that branch of engineering education in one of America's leading technical schools. Notwithstanding the fact that the courses in pure mathematics then given there were rigid and even severe, the students, as a rule, had no idea of how properly to apply the knowledge they had accumulated; nor did they know what the mathematical terms employed really meant. It was necessary for the writer not only to teach his own branch, but also to supplement the students' knowledge of pure mathematics by explaining such things as limits, differential coeffi-

cients, total and partial differentials, and maxima and minima.

Throughout the entire course in rational mechanics the writer either demanded from the students or gave them demonstrations of all difficult or important formulæ; and the students in explaining their blackboard work were repeatedly asked to state the denominations, not only of the equations as a whole, but also of their factors and component parts. The answers to such questions evidenced clearly whether the student had a true conception of the mathematical work he was doing, or whether he had merely memorized certain manipulations of quantities.

It was the writer's custom also to supplement as much as possible all analytical work by graphical demonstrations; and if he were to resume the teaching of mechanics, he would adhere to this method.

In teaching technical mechanics the writer followed only to a certain extent the manner of instruction just described; for by the time his students had reached the technical studies, they were so well drilled and weeded out that constant quizzing on fundamentals was no longer necessary; nevertheless the question, "what is the denomination of that equation or of that quantity," was one that was very likely to be asked any student who gave his demonstrations haltingly or who evidenced at all a lack of conception of the principles involved.

In the writer's opinion, the manner of teaching pure mathematics to engineering students should differ materially from that usually employed in academic courses; for while in the latter case it suffices if the instructors be good mathematicians, in the former they should also be engineers, and should have taught, or at least should have studied specially, both rational and technical mechanics.

Some institutions still adhere to the antiquated custom of teaching pure mathematics by lectures. This method has always appeared to the writer to be perfectly absurd; for the primary benefit to be obtained from the study of mathematics is mental training; and the student can get this only by severe effort, and not by having another man's mind do the reasoning for him. Midnight oil and the damp towel are for most students necessary accessories to the courses in pure mathematics.

The writer believes that the only legitimate lectures in pure-mathematical courses for engineering students are as follows:

First: A short opening lecture to outline the work that is to be covered in the course and to explain how best to study the subject.

Second: Frequent informal talks to indicate the application of the mathematics studied to engineering practise, to explain clearly the meaning of all equations, factors and terms, and to show the true *raison d'être* of all that is being done.

Third: A concluding lecture in the nature of a résumé to call attention to what has been accomplished during the entire course and to the importance thereof.

Fourth: Personal and forcible lectures to lazy students so as to give them clearly to understand that they must either study harder or drop out of the class.

All mathematical work done by engineering students should be so thorough and complete that the subject shall be almost as much at command as the English language or the four simple rules of arithmetic. Only such thorough knowledge will enable the engineer to use mathematics readily as a tool, rather than as a final resource to be employed solely in extreme need.

Analytical geometry should be taught graphically as well as analytically in order that the student shall comprehend it fully and shall realize that the work is real and tangible and that the equations represent

lines, surfaces, and volumes, and are not the results of mere gymnastics. A knowledge of the graphics of analytical geometry is especially valuable in mechanical work, in the investigation of earth pressures, in suspension, bridge work, and in many other lines of engineering.

The proper conception of the meaning of the calculus is rarely carried away by the student. He knows the rules and can perform the operations, but their significance is beyond him; consequently he does haltingly and bunglingly the original work which facility in the use of the calculus should enable him to perform easily and well. This state of affairs is a crying evil which should be corrected in all schools that aim to give first class engineering courses.

Descriptive geometry is of very large value in the preparation of drawings; but, in addition, a thorough knowledge of it greatly aids in the conception of an object in space, and, consequently, is of large assistance in the evolution of original designs. A knowledge of it prior to the study of the courses in pure mathematics assists materially in the conception of what the latter really mean; consequently descriptive geometry should be one of the earliest courses in an engineering curriculum.

A sound knowledge of mechanics, the foundation of engineering, is impossible without a thorough understanding of mathematics. It is true that mechanics may be learned by rote or by so-called common-sense methods; but the "rule of thumb" or "pocket-book" engineer never rises to noticeable heights. Such an engineer almost invariably fails at the critical moment, when a decision must be supported by fundamental principles. It is true that the actual use of analytical geometry, calculus, least squares, or even higher algebra and spherical trigonometry, is rare in the practise of most engineers; but an engineer's grasp of technical work depends upon his knowledge of these subjects; and it is generally conceded that a heavy structure can not be continuously supported on a weak foundation.

Mathematics higher than the calculus is of small value to the engineer, except possibly as a training for the mind; but the writer is of the opinion that any such further study of mathematics is a detriment rather than a help, in that it tends to a desire to reduce all work to mathematical calculation and thus to weaken the judgment. In other words, excess of mathematical development sometimes produces an unpractical engineer.

Most graduate engineers immediately after leaving their *alma mater* drop forever the study of mathematics, both pure and applied, except in so far as they are forced to use them by their professional work. No greater mistake than this can be made, for it takes very few years of non-use of these subjects to cause one to forget them utterly. Every young engineer should make it a point to devote a certain portion of his time to the reviewing of the mathematical studies of his technical course so as never to become rusty in them; and the writer believes that it is the duty of every professor of mathematics and mechanics to impress this fact continually upon the minds of his students, even up to the very day of their graduation.

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FROM THE STANDPOINT OF THE PROFESSOR OF ENGINEERING

When I come to think of what the Mathematical Society has brought upon itself, I fear that it may feel something like the football when it is kicked back and forth upon the field. On the one hand we have the trade-school element demanding more knowledge of rules and, on the other, the

engineer demanding more knowledge of principles. No fair discussion of this subject can be had without considering for a moment the conditions and definition of engineering itself. The most common definition was promulgated more than half a century ago by Thomas Tredgold, to the effect that civil engineering, which was the only branch of engineering then known, so the definition may be considered as being general, that "civil engineering is the art of directing the great sources of power in nature to the use and convenience of man." I should say that "civil engineering to-day is the art *and science* of directing the great sources of power in nature to the use and convenience of man," and from that standpoint I am willing to discuss the question as to how much and how far mathematical instruction should enter.

If engineering is merely an art, then mathematics as a science has no place in the training of the engineer, but if engineering is a science, then mathematics has a place. Engineering stands to-day in the act of rising to the status of a science, but is still hampered by the tradesman. On the one hand, we have the demand that the student's training be such as primarily to make him useful to some one to-morrow; and, on the other side, that it make him useful to the world perhaps ten years hence. The two requirements are inconsistent and do not belong together. One is that of the trade school, and many should not go farther than that because they have not the mental capacity, and the other is the demand of the profession into which a smaller number are qualified to enter. The trade school has caused most of the trouble with the teaching of mathematics because those who are products of the trade school have no use for mathematics as a science. The complaint about the teaching of mathematics does not come from engineers; they are ready to use mathematics as a science. In civil engineering it is fortunate that the profession has developed along lines laid down by Rankine rather than by Trautwine. Both have had their use, but one of them produced the scientist and the other produced the tradesman.

It is maintained in the institution which I have the honor to represent that they who would teach engineering must practise it, and by analogy we might say that those who teach mathematics to engineers should themselves be engineers. It seems to me that a time may come when such a condition will be desirable, but let me say now that there are few engineers to-day who have had sufficient training in mathematics to teach it themselves, much less to tell mathematicians how it should be taught. We can perhaps judge of the deficiency of the student who comes to us, but my feeling is that the remedy is not a question of *what,* but of *how.* Men in my institution are sending us students well prepared in mathematics. Others do not seem to be so fortunate. Both are teaching the same subjects. We have to realize that the student himself is a factor in this question. Some students become mathematicians under any *one;* others would not under *any* one. To be taught mathematics properly, the point at which engineering minds must begin, is a long way back. I am inclined to think they must begin some generations before birth. The mathematics of grammar schools needs overhauling more than the mathematics of any other part of our educational system, and probably the mathematics of high schools stands next. The essential thing that we ask of mathematics is that it should develop the quantitative reasoning power, and the student must be able to think mathematically. If he has not acquired that, then he should drop out of engineering and take up a trade. It was mentioned by a previous speaker that a relatively small percentage

of the graduates from a certain engineering school were engaged in occupations in which mathematics was of importance. From a somewhat intimate acquaintance with the graduates of that institution, I may add that a much less proportion had sufficient mathematical training to take positions in which mathematics was an important requirement. Until recently, that college has stood for hardly more than a highly developed trade school, and it is not fair to cite its statistics as showing conditions of *engineering schools*. The director of that institution stated many years ago that he did not consider descriptive geometry necessary for mechanical engineers, and his students, having had their course in machine design in the junior year were frequently found taking their only course of descriptive geometry when seniors.

The question has been raised as to the increase of mathematics for entrance to engineering schools. My view of that is that it would not be wise to raise the requirements at this time. Cornell has, it is true, increased the requirements, but at the sacrifice of both physics and chemistry, and to my mind it is best that physics and chemistry be taught at the age of high school students, rather than analytics and trigonometry. If you can not do both it is better that the young mind have impressed upon it some physical science rather than encounter the more abstract demands of mathematics. In the training of students in mathematics I would wipe out formulæ. We want principles. There is generally taught too much of the formula, as that is what the trade school has demanded. Some have objected to the statement that mathematics should be a tool. To my mind it is certainly an instrument. It is one of the things that the engineer must use, and in order that he may use it, he must be sufficiently familiar with it, so that it will respond to his use when he desires it. The question of election in mathematics has been suggested. I am certainly favorable to elections in that subject, but I question the advisability of such opportunity in any subject for the ordinary student, before the fourth year. My own observation leads me to conclude that very few students are able to elect intelligently before that time. The remarks relative to the employment of inexperienced instructors instead of competent professors show a fault to lie with the heads of the various departments themselves. If they are willing to accept, for the purpose of instructing students, the men who have been unable to find positions elsewhere, and employ only such as will work for seven to nine hundred dollars per year, the unsatisfactory results are their own fault. The responsible parties, the trustees and regents of educational institutions, will furnish what is shown to be necessary. If it is necessary that you have better men, then say so and get them, but if you are satisfied with what you now have, then you can expect to see decorative cornices and stained glass windows, rather than intellect and culture, the characteristics of our universities.

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It may save time to state briefly at the beginning my thought on what is needed in the teaching of mathematics to engineering students. It seems to me that, outside of the general cultural and developmental purpose of the study of mathematics, the instruction of engineering students may be discussed under three different phases, which for want of better terms may be named: (1) theory, (2) practise, (3) philosophy; that successful teaching of mathematics to engineering students depends upon giving the right

relative proportion or emphasis to these three phases of instruction; that the content of the instruction, within the limits of present usage in engineering schools, is of minor importance; that thoroughness is essential, and that it is better to cut down the extent of the matter gone over if thereby a more thorough grasp of the subject is secured; and that the instructor must always keep in mind that he is training an average boy of average preparation with a view to using mathematical principles and methods of attack and mathematical operations and conceptions in the mastery of his engineering studies and in the treatment of the varied problems which will arise in his later engineering experience.

The great mass of our engineering students, like the great mass of our engineers, are not mathematical geniuses. In the discussion of the subject we must keep ever in mind that the average engineering student is not of strong mathematical bent. Many of those with only mediocre mathematical ability make successful engineers, and the student of strong mathematical turn may lack in some direction or may have a disproportionate measure of the importance of his analytical powers and drop behind his less mathematical classmate. I want to make a plea for the average student, the boy whose analytical powers have to be encouraged and developed. The methods of presentation must be made elastic enough to include this great class of students, or we shall fail to do our duty as teachers.

I have mentioned three phases in the presentation of mathematical subjects. These may be considered in order. It must be understood that these phases are not mutually exclusive.

1. *Theory.*—Analysis, demonstration and the general derivation and presentation of mathematical principles. The derivation and exposition of mathematical principles and operations and the appreciation of mathematical concepts are universally accepted as important elements in the education of an engineer. The use of mathematical forms of attack, the training in processes of reasoning, the formation of logical habits of thought, are hardly secondary in importance. And yet much less emphasis is placed on formal demonstration and reasoning than formerly—frequently this element is overlooked or treated in a slipshod way. The student comes to feel that he is after facts and that the derivation and proof of principles involves useless effort—he is willing to accept their authenticity. It may be that years ago our instructional methods carried formal processes to an extreme and that as a result mathematical work became meaningless lingo or memorized facts to many students. This does not furnish argument for the abandonment of training in formal reasoning. For the young mind, practise in analysis, in formal demonstration is illuminating and developing. Even the repetitive forms of analysis in the old-time mental arithmetic had great mathematical educational value. The speaker feels that in the effort to avoid barren formalism the pendulum has swung too far the other way, and that both in high school and in technical school, and in the applied engineering subjects as well, the training in analytical methods and formal processes is weak. He believes that good results would follow putting greater emphasis on this phase of instruction than now seems to be the trend.

2. *Practise.*—The use and applicability of mathematical principles and processes in the solution of problems, drill on these principles, and the acquisition of facility in their use. To the average student the working of examples is illuminating. Without it the concept is but vaguely comprehended, the derivation only faintly

understood, the process may seem merely verbal legerdemain. Properly used, this phase of mathematical instruction is of great advantage to the student of average mathematical ability. It opens up the view; it clears away uncertainties; it fixes principles and concepts; it gives life to the subject. The problems used should be within the field of the students' experience and comprehension and may well bear some relation to his future work, both in the engineering class-room and beyond. And the second part of this heading is not less important. Mathematics is a tool for the engineering student, and he must acquire facility in its use. This does not mean that the instructor should attempt to make him a finished calculator or an expert workman—time is too short—but mathematical principles and processes must be more to the student than a vague something which he recognizes when his attention is directed thereto. Instead, he must have a mastery of at least the fundamentals and he must be able to use such principles and processes in his later studies without having to divert his attention and energy too much from the engineering features involved. To acquire this facility requires drill and repetition, and this drill must constitute a part of the mathematical training of the engineering student. The multiplication table had to be learned, and many other important things have to be acquired in the same way.

But it seems that this important side of instruction may be abused. The student who thinks that to accept facts and work problems is sufficient and the instructor who thinks that illustrations and practise work alone constitute mathematical training or that mere laboratory methods suffice are greatly mistaken. The mere substitution in formulas is only rule-of-thumb work, so much decried in engineering; and the mechanic who knows how to use tools, and no more, is not an engineer. There must be a direct connection with the theory and the philosophy of the subject to make the practise side serve its proper purpose. In teaching mathematics years ago, expressions of approval came to me because I was so "practical," but the underlying purpose of the practical part was not always understood, though this lack of understanding did not affect the results of the method. Inside the "sugar coating" there should always be a principle to fix, a concept to illumine, a process to exemplify, a derivation to expound. There seems to be a tendency among some to overdo this side of the work to the detriment of the first side. While the practise feature is a valuable auxiliary in mathematical instruction, it should never be the leading motive. Student and instructor alike should recognize this.

3. *Philosophy of the Subject.*—The basis on which the science rests, the underlying meaning of the mathematical processes used, a philosophical study of the method of treatment and of the concepts used, their connection with related things. This is difficult to discuss in a general way, and of course this phase is intimately connected with the first and second. To my mind this phase should not be neglected. It must be apportioned according to the ability of the student. An understanding of the philosophy of the subject will widen his field of view and lessen the chances of error. The better grasp of the meaning will be advantageous. Its presentation involves difficulties, and text-books generally disregard it. It must not be overemphasized, as is illustrated by the treatment in a recent text-book in applied mathematics, where it is used largely to the exclusion of analysis and demonstration.

Effective methods in mathematical subjects involve, then, the skillful selection in proper proportion from these three phases,

and the best teacher will make for himself the best selection. The derivation and elucidation of mathematical principles, facility in their use and application, and an understanding of the basis on which principles and methods rest are all essential. A good text-book—one properly proportioned—aids greatly in the work of instruction. However, it is the teacher on whom reliance is placed in the end, and for the student of average mathematical ability the teacher's influence constitutes a large element. It is highly advantageous for the teacher to have a fair knowledge of the applications of mathematics which the student will make in later work and to have sympathy and interest in such work. Let us also emphasize the importance of having the best of teachers for mathematical instruction.

Let me add to this that it is my belief, growing stronger after many years of observation, that the average engineering student gets relatively little from lectures on mathematical subjects; that many instructors talk too much themselves; that the student must have the opportunity to express himself and must be required to use the mathematical language and to try his own skill, and this in other than formal quizzes; and that recitation and drill work are essential factors in giving training to this average student.

Little can be said in the time at my disposal on the ground which should be covered in mathematical instruction. Two classes of matter are studied: (1) fundamental principles forming the skeleton of the work, and (2) the more complicated topics, involving further detail and insight. There will be little difference of opinion on the first class. There will be more on the second. I have found in the teaching of mechanics and of various engineering subjects that certain topics and methods not ordinarily given in mathematical instruction may advantageously be used in the presentation of the work. The teacher of thermo-dynamics or of electro-dynamics has other topics to suggest, and still other topics will come from other sources. Not all of these may be allowed. In fact, it makes little difference what particular topics are included so long as the student has thorough training in some of the more complex work. The difficulty of giving instruction in complex work lies not so much in the time required, as in the obstacle that the concepts lie beyond the student's experience and that he is not ready to comprehend their meaning. If he had the opportunity to study these topics after he has reached the subject in which they are to be used, or if he could go back over a part of mathematics after his study has taken him into their field of application, as indeed his instructor has done for himself, the result would be more satisfactory. All these limitations must be considered in choosing the ground to be covered in mathematical instruction.

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GRADUATE SCHOOL OF HOME ECONOMICS

The Graduate School of Home Economics held its second session at Cornell University, July 13–24. Representatives were present from eleven states and Canada. It is the purpose of this school to consider some of the results of the latest investigations in science, economics and art with their applications to work in home economics; the program, therefore, covered a wide range of subjects.

Practical demonstrations of household appliances were given by Misses Van Rensselaer and Rose, of the department of home economics in Cornell University. "Biology in its Relation to Home Economics" was discussed by Dr. J. G. Needham, of Cornell University; "Political Economy in its Relation to Home Economics" was discussed by Professor Fetter and Professor Kemmerer, of the